Lecture 2

Basic Radiometric Quantities. The Beer-Bouguer-Lambert law. Concepts of extinction (scattering plus absorption) and emission. Schwarzschild's equation.

Objectives:

1. Basic introduction to electromagnetic field: Definitions, dual nature of electromagnetic radiation, electromagnetic spectrum.

2. Basic radiometric quantities: energy, intensity, and flux.

3. The Beer-Bouguer-Lambert law. Concepts of extinction (scattering + absorption) and emission. Optical depth.

4. Simple aspects of radiative transfer: Schwarzschild's radiative transfer equation.

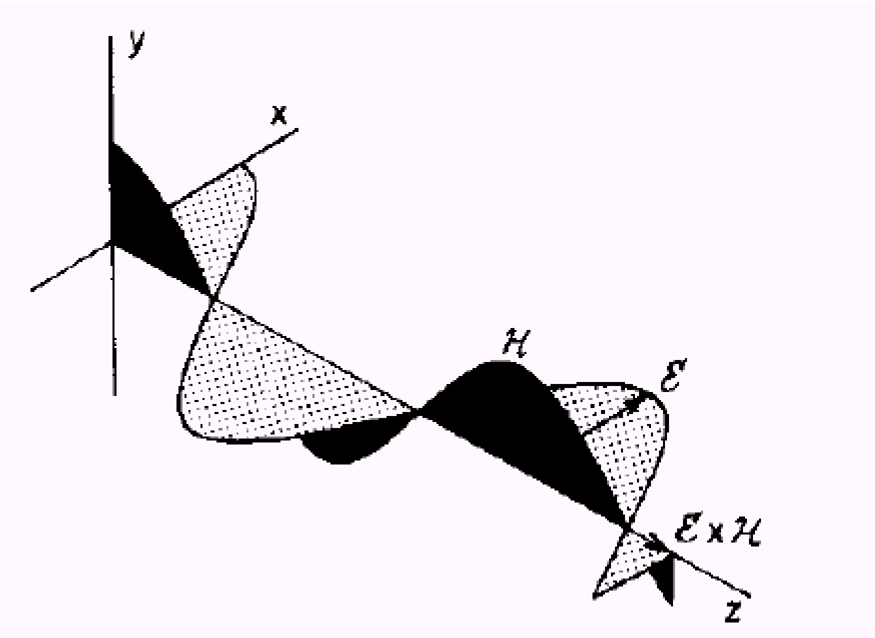
Required reading:

L02: 1.1, 1.4

1. Basic introduction to electromagnetic field.

Electromagnetic radiation is a form of transmitted energy**.** ***Electromagnetic radiation*** is so-named because it has electric and magnetic fields that simultaneously oscillate in planes mutually perpendicular to each other and to the direction of propagation through space.

- Radiation properties: Intensity, Phase, Polarization.
- Radiation depends on: Frequency, Space, Time, Direction.



- Electromagnetic radiation exhibits the **dual nature**:

wave properties and particulate properties.

➢ **Wave nature of radiation:** radiation can be thought of as **a traveling wave.**

Electromagnetic waves are characterized by **wavelength (or frequency)** and **speed**.

- The speed of light in a vacuum: c = 2.9979×10^8 m/s $\cong$ 3.00×10^8 m/s

The Spectrum

- **Wavelength** λ related to frequency $\tilde{\nu}$ and speed of light c by

$$\lambda = \frac{c}{\tilde{\nu}} \qquad c = 2.998 \times 10^8 \ \mathrm{m/s}$$

- **Wavenumber** ν is number of waves in a given length (usually 1 cm) and is proportional to frequency:

$$\nu = \frac{1}{\lambda} = \frac{10000 \ \mathrm{cm}^{-1} \ \mu\mathrm{m}}{\lambda}$$

Example: 8-12 μm atmospheric window is 833-1250 cm^{-1}.

We will mainly use wavelength and wavenumber. Wavenumber is used especially for molecular absorption spectroscopy.

Wavelength, λ, is the distance between two consecutive peaks or troughs in a wave.

Frequency, $\tilde{\nu}$, is defined as the number of waves (*cycles*) per second that pass a given point in space.

Wavenumber, ν , is defined as a count of the number of wave crests (or troughs) in a given unit of length.

Relation between λ, ν **and** $\tilde{\nu}$:

$$\boxed{\nu = \tilde{\nu}/c = 1/\lambda} \qquad [2.1]$$

UNITS:

Wavelength units: LENGTH,

Angstrom (A) : $1\ A = 1x10^{-10}$ m;

Nanometer (nm): $1\ nm = 1x10^{-9}$ m;

Micrometer (µm): $1\ \mu m = 1x10^{-6}$ m;

Frequency units: unit cycles per second 1/s (or s^{-1}) is called hertz (abbreviated Hz)

Wavenumber units: $LENGTH^{-1}$ (often in cm^{-1})

Spectrum of electromagnetic radiation

Table 2.1 Relationships between radiation components studied in this course.

Name of spectral region	Wavelength region, µm	Spectral equivalence
Solar	0.1 - 4	Ultraviolet + Visible + Near infrared = Shortwave
Terrestrial	4 - 100	Far infrared = Longwave
Infrared	0.75 - 100	Near infrared + Far infrared
Ultraviolet	0.1 - 0.38	Near ultraviolet + Far ultraviolet = UV-A + UV-B + UV-C + Far ultraviolet
Shortwave	0.1 - 4	Solar = Near infrared + Visible + Ultraviolet
Longwave	4 - 100	Terrestrial = Far infrared
Visible	0.38 - 0.75	Shortwave - Near infrared - Ultraviolet
Near infrared	0.75 - 4	Solar - Visible - Ultraviolet = Infrared - Far infrared
Far infrared	4 - 100	Terrestrial = Longwave = Infrared - Near infrared
Thermal	4 - 100	Terrestrial = Longwave = Far infrared

NOTE: $$\nu[cm^{-1}] = \frac{10000 cm^{-1} \mu m}{\lambda[\mu m]}$$

EXAMPLE: 8-12 µm atmospheric window is 833-1250 cm^{-1}

➢ Particulate nature of radiation:

Radiation can be also described in terms of particles of energy, called **photons**.

The energy of a **photon** is given by the expression:

$$\boxed{E_{photon} = h\,\tilde{\nu} = h\,c/\lambda = h\,c\nu} \qquad [2.2],$$

where h is Plank's constant ($h = 6.6256 \times 10^{-34}$ J s).

NOTE: Plank's constant h is very small!

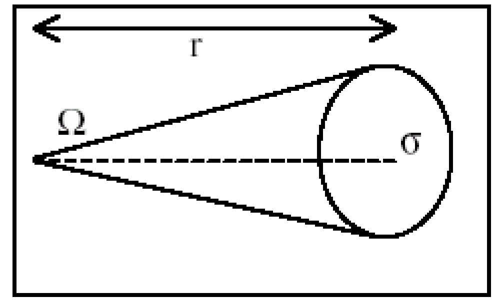
- Eq. [2.2] relates energy of each photon of the radiation to the electromagnetic wave characteristics ($\tilde{\nu}$ and λ).

- The quantized nature of light is most important when considering absorption of radiation by atoms and molecules.

2. Basic radiometric quantities.

Solid angle is the angle subtended at the center of a sphere by an area on its surface numerically equal to the square of the radius

$$\Omega = \frac{\sigma}{r^2}$$



UNITS: of a solid angle = steradian (sr)

A differential solid angle can be expressed as

$$d\Omega = \frac{d\sigma}{r^2} = \sin(\theta)\, d\theta d\phi \;,$$

using that a differential area is

$d\sigma = (r\, d\theta)\, (r \sin(\theta)\, d\phi)$

Change of variables: use (μ,ϕ) for directions, $\mu = \cos\theta \quad d\Omega = d\mu d\phi$

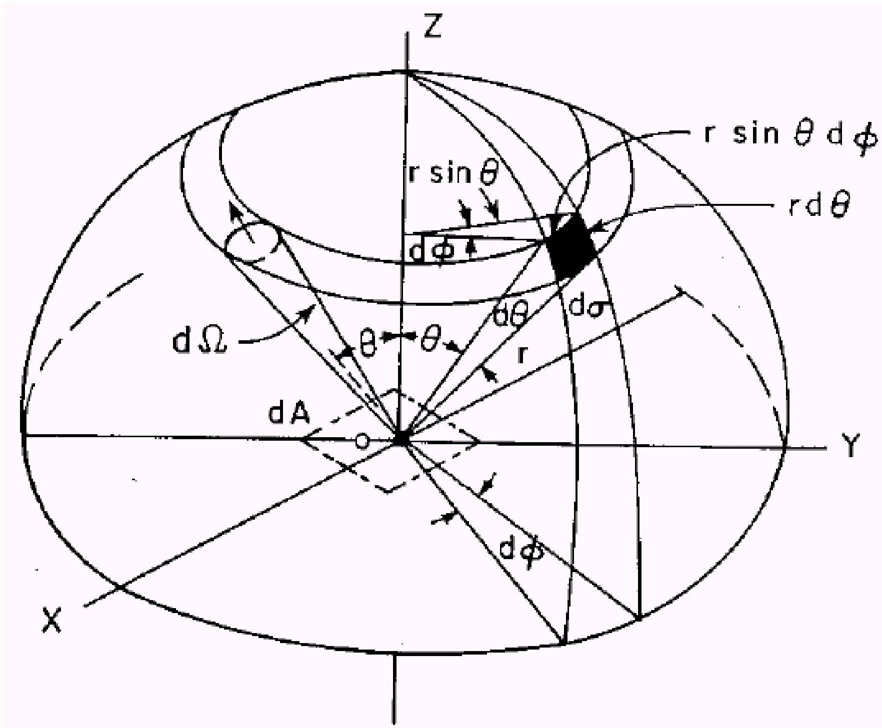
Solid angle for all directions: $\Omega = \int_0^{2\pi} \int_{-1}^{+1} d\mu d\phi \;= 4\pi$

$\mu = 1$ $(\theta = 0)$ is towards zenith, $\mu = -1$ $(\theta = 180)$ is towards nadir,

$\mu = 0$ $(\theta = 90)$ is towards horizon.

Warning: sometimes we will keep $\mu > 0$ even for downwelling.

Illustration of differential solid angle $d\Omega$ in spherical coordinates for cone of radiation around zenith angle θ and azimuth angle ϕ.



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EXAMPLE: Solid angle of a unit sphere = 4π

EXAMPLE: What is the solid angle of the Sun from the Earth if the distance from the Sun from the Earth is $d = 1.5 \times 10^8$ km and Sun's radius is $R_s = 6.96 \times 10^5$ km.

$$\Omega = \frac{\pi R_s^2}{d^2} = 6.76 x 10^{-5}\, sr$$

Intensity

Intensity (or radiance) is defined as radiant energy in a given direction per unit time per unit wavelength (or frequency) range per unit solid angle per unit area perpendicular to the given direction:

$$I_\lambda = \frac{dE_\lambda}{\cos(\theta)\, d\Omega\, dt\, dA\, d\lambda} \qquad [2.3]$$

I_λ is referred to as **monochromatic** intensity.

- Monochromatic does not mean at a single wavelengths λ, but in a very narrow (infinitesimal) range of wavelength $\Delta\lambda$ centered at λ.

NOTE: *same name*: intensity = specific intensity = radiance

UNITS: from Eq.[2.3]:

(J sec^{-1} sr^{-1} m^{-2} μm^{-1}) = (W sr^{-1} m^{-2} μm^{-1})

Properties of Intensity

- In general, intensity is a function of the coordinates ($\vec{r}$), direction ($\vec{\Omega}$), wavelength (or frequency), and time. Thus, it depends on seven independent variables: three in space, two in angle, one in wavelength (or frequency) and one in time.
- Intensity, as a function of position and direction, gives a complete description of the electromagnetic field.
- If intensity does not depend on the direction, the electromagnetic field is said to be **isotropic.** If intensity does not depend on position the field is said to be **homogeneous**.

Flux or irradiance

Flux (or irradiance) is defined as radiant energy in a given direction per unit time per unit wavelength (or frequency) range per unit area perpendicular to the given direction:

$$F_\lambda = \frac{dE_\lambda}{dt\,dA\,d\lambda} \qquad [2.4]$$

UNITS: from Eq.[2.4]:

(J sec^{-1} m^{-2} μm^{-1}) = (W m^{-2} μm^{-1})

From Eqs. [2.3]-[2.4]: $$F_\lambda = \int_\Omega I_\lambda \cos(\theta) d\Omega \qquad [2.5]$$

Thus, monochromatic **flux** is the integration of normal component of monochromatic **intensity** over some solid angle.

- Monochromatic **upwelling (upward) hemispherical flux** on a horizontal plane is the integration of normal component of monochromatic **intensity** over the all solid angles in the upper hemisphere. Eq. [2.5] in spherical coordinates gives:

$$F_\lambda^{\uparrow} = \int_0^{2\pi} \int_0^{\pi/2} I_\lambda(\theta, \varphi) \cos(\theta) \sin(\theta) d\theta d\varphi = \int_0^{2\pi} \int_0^{1} I_\lambda(\mu, \varphi) \mu d\mu d\varphi$$

where $\boldsymbol{\mu = cos(\theta)}$

Example: The normal incidence spectral solar flux at 0.5 μm at the orbit of the Earth is 1962 W m^{-2} μm^{-1}. If the solar flux is converted to isotropic radiance with a reflective diffuser having 100% efficiency, what is the radiance?
Since isotropic radiance is independent of direction, the hemispheric flux is

$$F_\lambda = I_\lambda \int_0^{2\pi} \int_0^1 \mu \, d\mu d\phi \; = \pi I_\lambda$$

Therefore the radiance is $I_\lambda = (1962 \text{ W m}^{-2}\, \mu\text{m}^{-1})/\pi = 625 \text{ W m}^{-2}\text{ sr}^{-1}\, \mu\text{m}^{-1}$.

Monochromatic **net flux** is the integration of normal component of monochromatic **intensity** over the all solid angles (over 4π). **Net flux** for a horizontal plane is the difference in **upwelling and downwelling hemispherical fluxes**:

$$F_{net,\lambda} = F_\lambda^{\uparrow} - F_\lambda^{\downarrow} = \int_0^{2\pi} \int_{-1}^{1} I_\lambda(\mu, \varphi) \mu d\mu d\varphi$$

Spectral integration

- **Integral** (or total) **intensity** ***I*** and **flux** *F* are determined by integrating over the wavelength the monochromatic intensity and flux, respectively:

$$I = \int_0^\infty I_\lambda d\lambda \qquad F = \int_0^\infty F_\lambda d\lambda$$

- Intensities and fluxes may be *per wavelength* or *per wavenumber*. Since intensity across a spectral interval must be the same, we have $I_\lambda d\lambda = I_\nu d\nu$ and thus

$$I_\nu = I_\lambda \left|\frac{d\lambda}{d\nu}\right| = I_\lambda \frac{1}{\nu^2} = I_\lambda \lambda^2$$

EXAMPLE: Convert between radiance in *per wavelength* to radiance *per wavenumber* units at λ = 10 μm. Given $\boldsymbol{I_\lambda}$ = 9.9 W m^{-2} sr^{-1}μm^{-1}. What is $\boldsymbol{I_\nu}$?

$\boldsymbol{I_\nu}$ = (9.9 W m^{-2} sr^{-1}μm^{-1}) (10 μm) (10^{-3} cm) = 0.099 W m^{-2} sr^{-1}(cm^{-1})$^{-1}$

Radiance vs. Flux

Constancy of intensity: If radiation is not interacting with matter then radiance is constant along a ray.
Solar flux depends on distance from sun (inverse square law):

$$F = F_{\oplus}\frac{r_{\oplus}^2}{r^2}$$

Flux decreases with distance squared because area of a sphere centered on the sun grows as r^2 and power crossing a sphere must be constant.
But intensity of solar radiation is constant because $I = F/\Omega$ and solid angle subtended by Sun decreases as $1/r^2$.

From an extended source, both radiance and flux are constant for a transparent medium. For example, the upward hemispheric flux from the moon's surface is constant with height because the solid angle subtended by the surfaces remains 2π until the curvature of the moon becomes important.

The Beer-Bouguer-Lambert Law. Concepts of Extinction (scattering+ absorption) and emission.

- **Extinction** and **emission** are two main types of the interactions between an electromagnetic radiation field and a medium (e.g., the atmosphere).

General definition:

Extinction is a process that decreases the radiant **intensity**, while **emission** increases it.

NOTE: "same name": **extinction = attenuation**

Radiation is **emitted** by **all** bodies that have a temperature above absolute zero (O K) (often referred to as **thermal emission**).

- **Extinction** is due to **absorption** and **scattering**.

Absorption is a process that removes the radiant energy from an electromagnetic field and transfers it to other forms of energy.

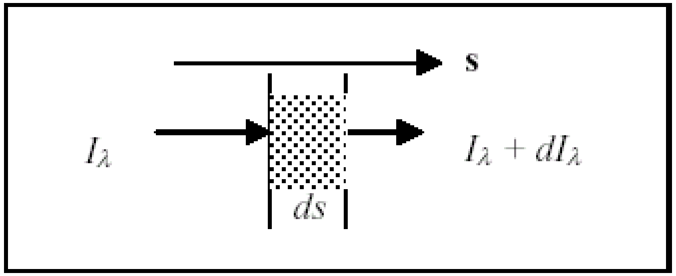
Scattering is a process that **does not** remove energy from the radiation field, but may redirect it.

NOTE: Scattering can be thought of as **absorption** of radiant energy followed by **re-emission** back to the electromagnetic field with negligible conversion of energy. Thus, scattering can remove radiant energy of a light beam traveling in one direction, but can be a "source" of radiant energy for the light beams traveling in other directions.

The fundamental law of extinction is the **Beer-Bouguer-Lambert law,** which states that the extinction process is linear in the intensity of radiation and amount of matter, provided that the physical state (i.e., T, P, composition) is held constant.

NOTE: Some non-linear processes do occur as will be discussed later in the course.

Consider a small volume ΔV of infinitesimal length ds and area ΔA containing optically active matter. Thus, the change of intensity along a path ds is proportional to the amount of matter in the path.



For extinction: $$dI_\lambda = -\beta_{e,\lambda} I_\lambda ds \qquad [2.6]$$

For emission: $$dI_\lambda = \beta_{e,\lambda} J_\lambda ds \qquad [2.7]$$

where $\boldsymbol{\beta_{e,\lambda}}$ is the **volume extinction coefficient** (LENGTH^{-1}) and $\boldsymbol{J_\lambda}$ is the **source function**.

- In the most general case, the **source function** $\boldsymbol{J_\lambda}$ has emission and scattering contributions.

- Generally, the **volume extinction coefficient** is a function of position **s.**

  (Sometimes it may be expressed mathematically as $\beta_{e,\lambda}$ (s), but s is often dropped).

NOTE: Volume extinction coefficient is often referred to as the **extinction coefficient.**

Extinction coefficient = absorption coefficient + scattering coefficient

$$\boxed{\beta_{e,\lambda} = \beta_{a\lambda} + \beta_{s,\lambda}} \qquad [2.8]$$

NOTE: Extinction coefficient (as well as absorption and scattering coefficients) can be expressed in different forms according to the definition of the amount of matter (e.g., number concentrations, mass concentration, etc.) of matter in the path (see Lecture 4).

- **Volume** and **mass extinction coefficients** are most often used.

Mass extinction coefficient = volume extinction coefficient/density

UNITS: the mass coefficient is in unit area per unit mass (LENGTH^2 MASS^{-1}). For instance: (cm^2 g^{-1}), (m^2 kg^{-1}), etc.

If ρ is the density (mass concentration) of a given type of particles (or molecules), then

$$\boxed{\begin{aligned} \beta_{e,\lambda} &= \rho\beta^{*}_{e,\lambda} \\ \beta_{s,\lambda} &= \rho\beta^{*}_{s,\lambda} \\ \beta_{a,\lambda} &= \rho k_{\lambda} \end{aligned}} \qquad [2.9]$$

where the $\boldsymbol{\beta^{*}_{e,\lambda}}$ **;** $\boldsymbol{\beta^{*}_{s,\lambda}}$**,** and $\boldsymbol{k_{\lambda}}$ are the **mass extinction, scattering, and absorption coefficients**, respectively.

NOTE: L02 uses $\boldsymbol{k_{\lambda}}$ for both mass extinction and mass absorption coefficients!

Extinction Cross-section

The **extinction cross section** of a given particle (or molecule) is a parameter that measures the attenuation of electromagnetic radiation by this particle (or molecule). In the same fashion, **scattering and absorption cross sections** can be defined.

UNITS: the cross section is in unit area (LENGTH^2)

If N is the particle (or molecule) number concentration of a given type of particles (or molecules), then

$$\boxed{\begin{aligned} \beta_{e,\lambda} &= \sigma_{e,\lambda} N \\ \beta_{s,\lambda} &= \sigma_{s,\lambda} N \\ \beta_{a,\lambda} &= \sigma_{a,\lambda} N \end{aligned}} \qquad [2.10]$$

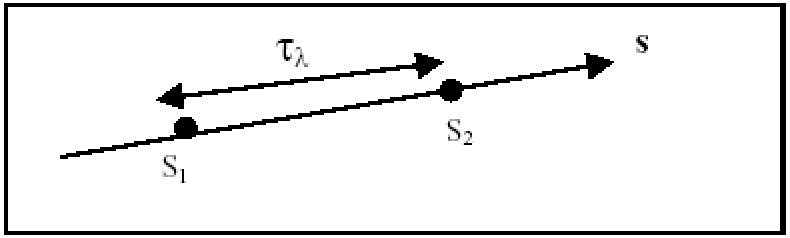
where $\boldsymbol{\sigma_{e,\lambda}}$, $\boldsymbol{\sigma_{s,\lambda}}$, and $\boldsymbol{\sigma_{a,\lambda}}$ are the extinction, scattering, and absorbing cross sections, respectively.

UNITS: Particle number concentration is in the number of particles per unit volume (LENGTH^{-3}).

Optical Depth

- **Optical depth** of a medium between points s_1 and s_2 is defined as

$$\tau_\lambda(s_2;s_1) = \int_{s_1}^{s_2} \beta_{e,\lambda}(s)ds$$



UNITS: optical depth is unitless.

NOTE: "same name": **optical depth = optical thickness = optical path**

- **If $\beta_{e,\lambda}$** (s) does not depend on position (called a homogeneous optical path), thus

$\beta_{e,\lambda}(s) = <\beta_{e,\lambda}>$ and $\tau_\lambda(s_2;s_1) = <\beta_{e,\lambda}>(s_2 - s_1) = <\beta_{e,\lambda}> s$

For this case, the **Extiction law** can be expressed as

$$\boxed{I_\lambda = I_0 \exp(-\tau) = I_0 \exp(-<\beta_{e,\lambda}> s)} \qquad [2.11]$$

Optical depth can be expressed in several ways:

$$\tau_\lambda(s_1;s_2) = \int_{s_1}^{s_2} \beta_{e,\lambda} ds = \int_{s_1}^{s_2} \rho \beta^*_{e,\lambda} ds = \int_{s_1}^{s_2} N \sigma_{e,\lambda} ds \qquad [2.12]$$

- If in a given volume there are several types of optically active particles each with $\boldsymbol{\beta^i_{e,\lambda}}$, etc., then the optical depth can be expressed as:

$$\tau_\lambda = \sum_i \int_{s_1}^{s_2} \beta^i_{e,\lambda} \, ds = \sum_i \int_{s_1}^{s_2} \rho_i \beta^{*i}_{e,\lambda} ds = \sum_i \int_{s_1}^{s_2} N_i \sigma^i_{e,\lambda} ds \qquad [2.13]$$

where $\boldsymbol{\rho_i}$ and $\boldsymbol{N_i}$ is the mass concentrations (densities) and particles concentrations of the ***i***-th species.

4. Simple aspects of radiative transfer.

Let's consider a small volume ΔV of infinitesimal length ds and area ΔA containing optically active matter. Using the **Extinction law**, the change (loss plus gain due to both the thermal emission and scattering) of intensity along a path ds is

$$dI_\lambda = -\beta_{e,\lambda} I_\lambda ds + \beta_{e,\lambda} J_\lambda ds$$

Dividing this equation by $\beta_{e,\lambda} ds$, we find

$$\boxed{\frac{dI_\lambda}{\beta_{e,\lambda} ds} = -I_\lambda + J_\lambda} \qquad [2.14]$$

Eq. [2.14] is the **differential equation of radiative transfer** **called** **Schwarzchild's equation.**

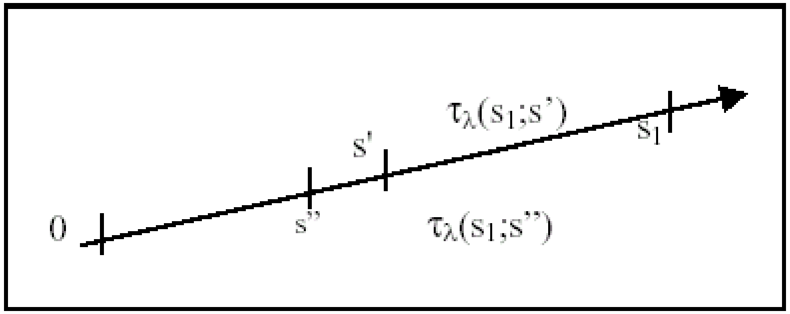
NOTE: Both I_λ and J_λ are generally functions of both position and direction.

The optical depth is

$$\tau_\lambda(s_1;s) = \int_s^{s_1} \beta_{e,\lambda}(s)\,ds$$

Thus

$$d\tau_\lambda = -\beta_{e,\lambda}(s)\,ds$$



Using the above expression for $d\tau_\lambda$, we can re-write Eq. [2.14] as

$$-\frac{dI_\lambda}{d\tau_\lambda} = -I_\lambda + J_\lambda$$

or as [2.15]

$$\frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - J_\lambda$$

These are other forms of the **differential equation of radiative transfer.**

Let's re-arrange terms in the above equation and multiply both sides by ***exp(-τλ)***. We have

$$-\frac{\exp(-\tau_\lambda)\,dI_\lambda}{d\tau_\lambda}+\exp(-\tau_\lambda)I_\lambda = \exp(-\tau_\lambda)J_\lambda$$

and (using that ***d[I(x)exp(-x)]=exp(-x)dI(x)-exp(-x)I(x)dx***) we find

$$-d\left[I_\lambda \exp(-\tau_\lambda)\right]= \exp(-\tau_\lambda)J_\lambda d\tau_\lambda$$

Then integrating over the path from **0** to $\mathbf{s_1}$, we have

$$-\int_0^{s_1} d\left[I_\lambda(s)\exp(-\tau_\lambda(s_1;s))\right]=\int_0^{s_1}\exp(-\tau_\lambda(s_1;s))J_\lambda d\tau_\lambda$$

and

$$-\left[I_\lambda(s_1)-I_\lambda(0)\exp(-\tau_\lambda(s_1;0))\right]=\int_0^{s_1}\exp(-\tau_\lambda(s_1;s))J_\lambda d\tau_\lambda$$

Thus

$$I_\lambda(s_1)=I_\lambda(0)\exp(-\tau_\lambda(s_1;0))-\int_0^{s_1}\exp(-\tau_\lambda(s_1;s))J_\lambda d\tau_\lambda$$

and, using $d\tau_{\lambda} = -\beta_{e,\lambda}(s)ds$, we have a **solution** of the **equation of radiative transfer** **(often referred to as the integral form of the radiative transfer equation):**

$$\boxed{I_{\lambda}(s_1) = I_{\lambda}(0)\exp(-\tau_{\lambda}(s_1;0)) + \int_0^{s_1} \exp(-\tau_{\lambda}(s_1;s))J_{\lambda}\beta_{e,\lambda}ds} \quad [2.16]$$

NOTE:

i) The above equation gives monochromatic intensity at a given point propagating in a given direction (often called an **elementary solution**). A completely general distribution of intensity in angle and wavelengths (or frequencies) can be obtained by repeating the elementary solution for all incident beams and for all wavelengths (or frequencies).

ii) Knowledge of the **source function** J_{λ} is required to solve the above equation. In the general case, the source function consists of thermal emission and scattering (or emission from scattering), depends on the position and direction, and is very complex. One may say that the radiative transfer equation is all about the source function.

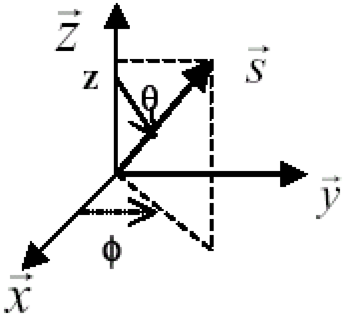
Plane-parallel atmosphere

- For many applications, the atmosphere can be approximated by **a plane-parallel model** to handle the vertical stratification of the atmosphere.

Plane-parallel atmosphere consists of a certain number of atmospheric layers each characterized by homogeneous properties (e.g., T, P, optical properties of a given species, etc.) and bordered by the bottom and top infinite plates (called boundaries).

- Traditionally, the **vertical coordinate** z is used to measure linear distances in the plane-parallel atmosphere:

$$z = s\cos(\theta)$$



where $\boldsymbol{\theta}$ denotes the angle between the upward normal and the direction of propagation of a light beam (or zenith angle) and $\boldsymbol{\phi}$ is the azimuthal angle.

Using ***ds = dz/cos(θ)***, the **radiative transfer equation** can be written as

$$\cos(\theta)\frac{dI_{\lambda}(z;\theta;\varphi)}{\beta_{e,\lambda}dz} = -I_{\lambda}(z;\theta;\varphi) + J_{\lambda}(z;\theta;\varphi)$$

Introducing the optical depth measured from the outer boundary downward as

$$\tau_{\lambda}(z_1;z) = \int_{z}^{z_1} \beta_{e,\lambda}(z)dz$$

and using $d\tau_{\lambda} = -\beta_{e,\lambda}(z)dz$ and $\boldsymbol{\mu = \cos(\theta)}$, we have

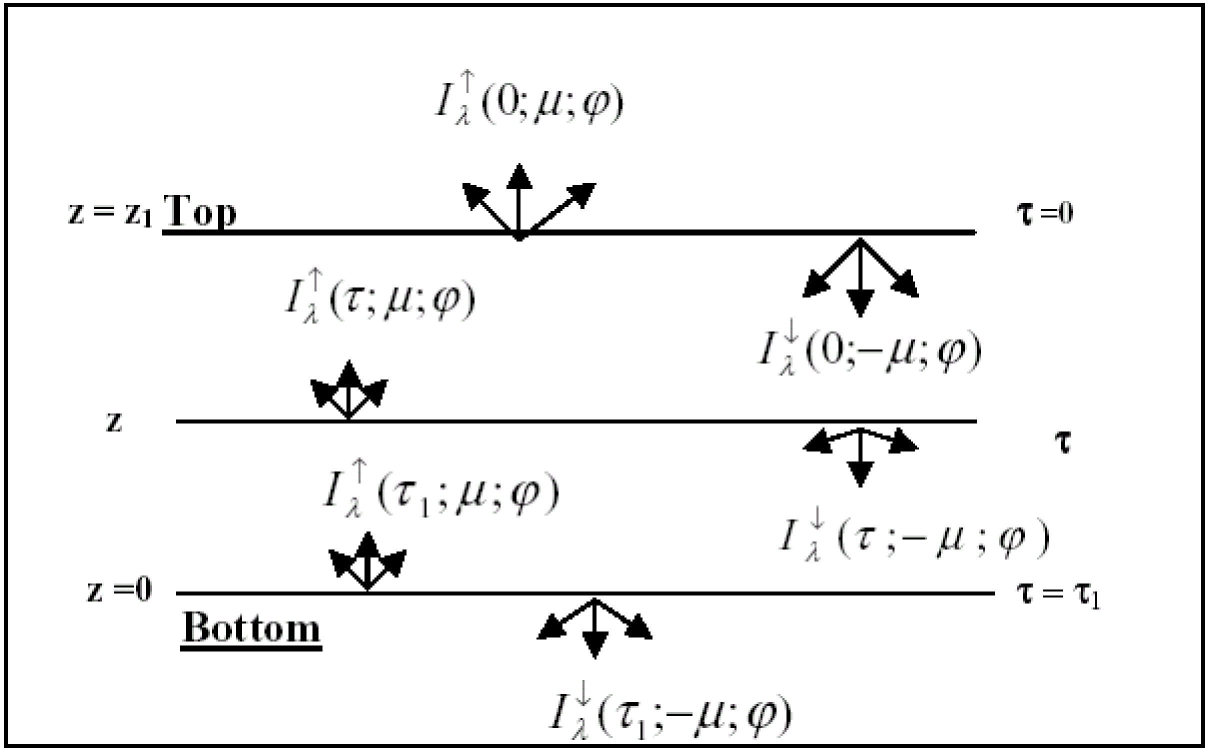
$$\boxed{\mu\frac{dI_{\lambda}(\tau;\mu;\varphi)}{d\tau} = I_{\lambda}(\tau;\mu;\varphi) - J_{\lambda}(\tau;\mu;\varphi)} \qquad [2.17]$$

Eq. [2.17] is the basic equation for the problem of radiative transfer in the plane-parallel atmosphere

- Eq.[2.17] may be solved to give the **upward (or upwelling) and downward (or downwelling) intensities** for a finite atmosphere which is bounded on two sites.

Upward intensity $I_\lambda^{\uparrow}$ is for $1 \geq \mu \geq 0$ (or $0 \leq \theta \leq \pi/2$);

Downward intensity $I_\lambda^{\downarrow}$ is for $-1 \leq \mu \leq 0$ (or $\pi/2 \leq \theta \leq \pi$)



Plane-parallel atmosphere.

NOTE: For downward intensity, μ is replaced by –μ.

The **radiative transfer equation** [2.17] can be written for **upward and downward intensities:**

$$\mu \frac{dI_\lambda^{\uparrow}(\tau;\mu;\varphi)}{d\tau} = I_\lambda^{\uparrow}(\tau;\mu;\varphi) - J_\lambda^{\uparrow}(\tau;\mu;\varphi) \qquad [2.18a]$$

$$-\mu \frac{dI_\lambda^{\downarrow}(\tau;-\mu;\varphi)}{d\tau} = I_\lambda^{\downarrow}(\tau;-\mu;\varphi) - J_\lambda^{\downarrow}(\tau;-\mu;\varphi) \qquad [2.18b]$$

A solution of Eq.[2.18a] gives a upward intensity in the plane-parallel atmosphere:

$$\boxed{\begin{aligned} I_\lambda^{\uparrow}(\tau;\mu;\varphi) &= I_\lambda^{\uparrow}(\tau_1;\mu;\varphi)\exp\left(-\frac{\tau_1-\tau}{\mu}\right) \\ &+ \frac{1}{\mu}\int_\tau^{\tau_1}\exp\left(-\frac{\tau'-\tau}{\mu}\right) J_\lambda^{\uparrow}(\tau';\mu;\varphi)\,d\tau' \end{aligned}} \qquad [2.19a]$$

and **a solution of Eq.[2.18b] gives a downward intensity in the plane-parallel atmosphere**:

$$\boxed{\begin{aligned} I_\lambda^{\downarrow}(\tau;-\mu;\varphi) &= I_\lambda^{\downarrow}(0;-\mu;\varphi)\exp\left(-\frac{\tau}{\mu}\right) \\ &+ \frac{1}{\mu}\int_0^{\tau}\exp\left(-\frac{\tau-\tau'}{\mu}\right) J_\lambda^{\downarrow}(\tau';-\mu;\varphi)\,d\tau' \end{aligned}} \qquad [2.19b]$$